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International Journal of Heat and Mass Transfer 47 (2004) 2469–2480
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Unsteady conjugate heat/mass transfer from a circular cylinder in laminar crossflow at low Reynolds numbers

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Abstract

Unsteady conjugate heat/mass transfer between a circular cylinder and a surrounding fluid flow has been analysed. Numerical investigations were carried out for the cylinder Reynolds numbers equal to 2 and 20 and different values of the Prandtl/Schmidt numbers. The heat/mass balance equations were solved numerically in cylindrical coordinates by the ADI finite difference method. The computations yield information regarding the influence of the parameters that characterize the coupling features of the conjugate heat/mass transfer, i.e. the physical properties ratios, on Nusselt/Sherwood numbers and cylinder dimensionless average temperature/concentration. The thermal/mass wake phenomenon were also studied.

1. Introduction

The usefulness and at the same time the necessity of a study dedicated to conjugate heat/mass transfer from an isolated circular cylinder in laminar crossflow can be twofold argued. First, the convective heat/mass transfer from and to an isolated circular cylinder is important in many engineering applications. Industrial processes in which this phenomenon plays an important role are: drying of different materials (textiles, veneer, paper and film materials), cooling of glass, plastics and industrial devices, from turbine blades to electronic circuits, and so on. Anemometry and chemical or radioactive contamination/purification are other important applications of this process.

Lange et al. [1] observed that ''compared with the isothermal flow past a cylinder, there are remarkably few publications on the numerical solutions of the heat transfer from a cylinder in crossflow''. Forced convection heat/mass transfer from a circular cylinder in laminar crossflow (we restricted the literature citation to articles similar to the present work) have been studied by Dennis et al. [2], Sucker and Brauer [3], Jain and Goel [4], Karniadakis [5], Chen and Weng [6], Yang et al. [7], Kurdyumov and Fernandez [8] and Lange et al. [1]. In the articles mentioned previously, the temperature/concentration of the cylinder is considered constant. This condition is fulfilled in practice if there is a source of energy/mass inside the cylinder. In many situations of practical importance there is no source of energy/mass inside the cylinder. In this case, the problem should be rewritten and solved as a conjugate one.

Secondly, the unsteady conjugate heat/mass transfer between a body of revolution and a surrounding fluid flow has been extensively studied considering spherical geometry for the body of revolution, [9–14]. Physical unsteady conjugate heat/mass transfer between a sphere and an ambient flow reveals some unusual aspects. The most important seems to be the mass/thermal wake phenomenon. It consists of the locally reverse of the transfer direction on the interface in the vicinity of the backward stagnation point. The occurrence and the evolution of this phenomenon are related to the values of the physical properties ratios of the two phases. The influence of the geometry of the body of revolution on this phenomenon was not analysed until now (according to our knowledge). This work is meant to be a first step in this direction.

The study of the unsteady conjugate heat/mass transfer from a circular cylinder in laminar crossflow is the subject of this paper. For motionless systems, the

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doi:10.1016/j.ijheatmasstransfer.2003.10.035

Nomenclature

c_P	heat capacity
C	concentration of the transferring species
d	cylinder diameter
D	diffusion coefficient of the transferring species
E	stream function
F	dimensionless stream function, $F = 2E/U_\infty d$
Pr	fluid phase Prandtl (Schmidt) number, $Pr = \nu/\alpha_f(D_f)$
Re	Reynolds number based on cylinder diameter, $Re = U_\infty d/\nu$
t	time
T	temperature
Z	dimensionless temperature/concentration defined by the relations, $Z_{c(f)} = \frac{T_{c(f)}-T_\infty}{T_{c,0}-T_\infty}$ or $Z_c = \frac{C_c - C_\infty \Xi}{C_{c,0}-C_\infty \Xi}$, $Z_f = \frac{C_f - C_\infty}{C_{c,0}/\Xi - C_\infty}$

Greek symbols

α	thermal diffusivity
Φ	λ_c/λ_f or $(D_c/D_f)/\Xi$
λ	thermal conductivity
ν	kinematic viscosity of the fluid phase
ρ	density
τ	dimensionless time or Fourier number, $\tau_{c(f)} = 4t\alpha_{c(f)}(D_{c(f)})/d^2$
ϖ	vorticity
ω	dimensionless vorticity, $\omega = \varpi d/2U_\infty$
Ξ	$(\rho_c c_{P,c})/(\rho_f c_{P,f})$ or Henry number

Subscripts

c	cylinder
f	environmental fluid
0	initial conditions
∞	large distance from the cylinder

solution of the unsteady conjugate heat transfer from a circular cylinder to a quiescent, infinite fluid medium can be viewed in [15]. This paper wishes to extend the analysis of Gröber et al. [15] on systems with fluid motion. The influence of the physical properties ratios on the heat/mass transfer rate is investigated in two hydrodynamic regimes: $Re = 2$ (steady laminar flow without separation) and $Re = 20$ (steady laminar flow with separation). For each Re number, two situations were considered: (1) $Pr(Sc)$ number constant such that $Re\,Pr(Sc) = 100$ and (2) $Pr(Sc)$ varying simultaneously with the physical properties ratios. The appearance and the development of the mass/thermal wake phenomenon is also investigated.

2. Basic equations

Let us consider an infinitely long horizontal circular cylinder of diameter d (considerably higher than the molecular mean free path of the surrounding fluid) placed in a vertical flow of an incompressible fluid having a free stream velocity U_∞ and constant temperature/concentration, T_∞/C_∞. We assume valid the following statements:

- the fluid is Newtonian and the flow is steady, laminar and axisymmetric;
- the effects of buoyancy and viscous dissipation are negligible;
- constant physical properties of the cylinder and surrounding medium;
- no emission or absorption of radiant energy;
- no phase change;
- no chemical reaction inside the cylinder or in the surrounding fluid;
- no pressure diffusion or thermal diffusion.

Under these assumptions, the governing non-dimensional equations, expressing the conservation of energy/chemical species, in cylindrical coordinates (r, θ) are

- inside the cylinder

$$\frac{\partial Z_c}{\partial \tau_c} = \frac{\partial^2 Z_c}{\partial r^2} + \frac{1}{r}\frac{\partial Z_c}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Z_c}{\partial \theta^2} \tag{1}$$

- in the surrounding fluid

$$\frac{\partial Z_f}{\partial \tau_f} + \frac{Re\,Pr}{2}\left(V_R \frac{\partial Z_f}{\partial x} + \frac{V_\theta}{r}\frac{\partial Z_f}{\partial \theta}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z_f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 Z_f}{\partial \theta^2} \tag{2}$$

The boundary conditions to be satisfied are

- cylinder center ($r = 0$)

$$Z_c = \text{finite} \tag{3a}$$

- interface ($r = 1$)

$$Z_c = Z_f, \quad \Phi\frac{\partial Z_c}{\partial r} = \frac{\partial Z_f}{\partial r} \tag{3b}$$

- free stream ($r = \infty$)

$$Z_f = 0.0 \tag{3c}$$

- symmetry axis ($\theta = 0, \pi$)

$$\frac{\partial Z_c}{\partial \theta} = \frac{\partial Z_f}{\partial \theta} = 0.0 \tag{3d}$$

The dimensionless initial conditions are

$$\tau_c = \tau_f = 0.0, \quad Z_c = 1.0, \quad Z_f = 0.0 \tag{4}$$

All velocities and lengths are scaled by U_∞ and by the cylinder radius $d/2$, respectively.

The assumptions practiced in this work are those usually employed in the analysis of the analogy between heat and mass transfer. Under these conditions, in this work, the dimensionless variable Z has a double signification, dimensionless concentration—dimensionless temperature.

The components of the velocity field (V_R, V_θ) were calculated by

$$V_R = -\frac{1}{r}\frac{\partial F}{\partial \theta} \tag{5}$$

$$V_\theta = \frac{\partial F}{\partial r} \tag{6}$$

where F is the dimensionless stream function for the steady, laminar flow around the cylinder. The values of $F = \psi + r\sin\theta$, ψ being the deviation from the uniform flow, were computed numerically by solving the Navier–Stokes equations, [16]:

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = -\omega \tag{7a}$$

$$\frac{Re}{2r}\left[\left(\frac{\partial \psi}{\partial r} + \sin\theta\right)\frac{\partial \omega}{\partial \theta} - \left(\frac{\partial \psi}{\partial \theta} + r\cos\theta\right)\frac{\partial \omega}{\partial r}\right] + \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r}\frac{\partial \omega}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2} = 0 \tag{7b}$$

with the boundary conditions, [16]:

- cylinder surface ($r = 1$)

$$\psi(1,\theta) = -\sin(\theta), \quad \frac{\partial \psi}{\partial r}(1,\theta) = -\sin(\theta) \tag{8a}$$

- symmetry axis ($\theta = 0, \pi$)

$$\psi = \omega = 0 \tag{8b}$$

- free stream ($r = \infty$)

$$\frac{\partial \psi}{\partial r} = \frac{\partial \omega}{\partial r} = 0.0 \tag{8c}$$

In any analysis, the asymptotic solutions play a specific role. For a conjugate problem, the following two situations deserve attention: (1) when Φ tends to zero, the gradients of the temperature/concentration in the surrounding fluid become negligible (the ambient medium becomes gradientless); (2) when Φ tends to infinite, the cylinder becomes gradientless. Case (1) is usually referred as "internal problem" while case (2) as "external problem". Both internal and external problem were solved in this work. The solution of the internal problem was computed by solving Eq. (1) with the boundary conditions (3a), (3d) and the modified boundary condition at $r = 1$,

$$Z_c = 0 \tag{9}$$

The solution of the external problem was computed by solving Eq. (2) with the boundary conditions (3c), (3d) and the modified boundary conditions at $r = 1$,

$$\frac{\partial Z_c}{\partial \tau_c} = -\frac{2}{\pi \Xi}\int_0^\pi \left.\frac{\partial Z_f}{\partial r}\right|_{r=1} d\theta \tag{10}$$

The important characteristic quantities of the heat/mass transfer from a circular cylinder in laminar cross-flow are

- the local Nu number;
- the instantaneous overall and fractional Nu numbers;
- the dimensionless average cylinder temperature/concentration.

The average cylinder dimensionless temperature/concentration was calculated by the relation:

$$\overline{Z}_c = \frac{2}{\pi}\int_0^1 \int_0^\pi Z_c r \, d\theta \, dr \tag{11}$$

The local dimensionless heat/mass transfer coefficient or Nusselt number based on the diameter of the cylinder is

$$Nu(\theta) = -\frac{2}{\overline{Z}_c}\left.\frac{\partial Z_f}{\partial r}\right|_{r=1} \tag{12}$$

The overall Nu numbers (Nu_c if $\Phi \leqslant 1$ and Nu_f if $\Phi > 1$) were calculated with the relations:

$$Nu_{c(f)} = -\Xi \frac{d \ln \overline{Z}_c}{d\tau_c} \tag{13}$$

or

$$Nu_{f(c)} = -(\Phi)\frac{2}{\pi \overline{Z}_c}\int_0^\pi \left.\frac{\partial Z_f}{\partial r}\right|_{r=1} d\theta \tag{14}$$

The fractional Nu numbers, i.e. the internal Nu number, Nu_{int}, and the external Nu number, Nu_{ext}, were computed as

$$Nu_{int} = -\frac{2}{\pi(\overline{Z}_c - \overline{Z}_{c,s})}\int_0^\pi \left.\frac{\partial Z_f}{\partial r}\right|_{r=1} d\theta \tag{15}$$

$$Nu_{ext} = -\frac{2}{\pi \overline{Z}_{c,s}}\int_0^\pi \left.\frac{\partial Z_f}{\partial r}\right|_{r=1} d\theta \tag{16}$$

where $\overline{Z}_{c,s}$ is the dimensionless surface average temperature/concentration

$$\overline{Z}_{c,s} = \frac{2}{\pi}\int_0^{\pi} Z_c|_{r=1}\,d\theta \tag{17}$$

3. Method of solution

The energy/mass balance equations and the Navier–Stokes equations were solved numerically. The radial coordinate r for the outer region was replaced by x through the transformation $r = \exp(x)$. As a result, the use of a constant discretization parameter for x made it possible to obtain a more dense mesh near the cylinder surface, where the gradients are large and where an accurate numerical approximation is needed. The finite difference method was used for discretization. The external boundary condition is assumed to be valid at a large but finite distance, r_∞, from the center of the cylinder. For the Re and Pr values employed in this study, $r_\infty = \exp(\pi)$ is a good approximation. The spatial meshes have 129×129 and 257×257 points in each phase.

The Navier–Stokes equations being uncoupled from the energy/mass balance equations can be solved independently of them. The algorithm employed is the nested defect-correction iteration, [17,18]. The method is well described in the references mentioned previously and it is not necessary to be reproduced here.

Eq. (1) was discretized with the central second order accurate finite difference scheme. In the fluid phase, the exponentially fitted scheme, [19], was used. The discrete parabolic equations were solved by the implicit ADI method. The time step was variable and changed from the start of the computation to the final stage. The initial and final values of the time step depend on the parameters values.

4. Results

The dimensionless equations (1), (2), and the boundary and initial conditions (3) and (4) depend on four dimensionless parameters: Re, Pr, Φ and Ξ. The first question discussed in this section is the selection of the numerical values of these parameters.

Two values of the cylinder Re number were used: $Re = 2$ and 20. In the range of the Re numbers investigated in the present work, the laminar flow around a circular cylinder has two regimes: steady flow without separation ($Re \leqslant 5$) and steady flow with two symmetric vortices behind the cylinder ($5 \leqslant Re \leqslant Re_{crt} \cong 46$). The numerical method used in this work to solve the Navier–Stokes equations converges for cylinder Re numbers greater than 20. However, for $30 < Re < 80$, the regime corresponding to the transition from a $2-D$ steady to a $2-D$ periodic wake, the wake behind a heated cylinder is very sensitive to the heat transfer. Over this Reynolds number range "the heat was never a passive contaminant", [20]. For this reason, Re values greater than 20 were not used in this work.

The impact of the hot-wire anemometry on the studies dedicated to forced convection heat transfer from a circular cylinder in laminar crossflow is huge. The majority of relations worked-out to calculate Nu are valid for $Pr = 0.71$ (Pr does not appear practically in these relations) (see, as example, [1,2,5,21]). In anemometry, the unsteady conjugate heat/mass transfer from a circular cylinder in laminar crossflow presents some interest only for the frequency response of the resistance thermometer. Applications of the conjugate transfer can be viewed in extraction from vegetal fibres, food industries, contamination or pollution problems and cooling processes.

The key parameters in any conjugate problem are those that characterize the coupling features of the transfer at the interface. In this work, these parameters were denoted by Φ and Ξ. Φ is the ratio (cylinder/surrounding medium) of two kinetic (transport) quantities; in the case of a heat transfer process Φ represents the ratio of the thermal conductivities while in the case of a mass transfer process Φ is the ratio (diffusivities ratio)/(Henry number). Ξ is the ratio (cylinder/surrounding medium) of two thermodynamic quantities; volume heat capacities ratio for a heat transfer process and Henry number (or distribution coefficient) for mass transfer phenomena. For brevity, Φ will be referred as the kinetic ratio while Ξ as the thermodynamic ratio.

We considered the cylinder as a solid body, porous or smooth. For a heat transfer process, if the environmental medium is a gas, the kinetic ratio varies between O(1) (for plastics) to O(10^3) (for metals) while the thermodynamic ratio takes values considerably greater than one. If the surrounding medium is a liquid, the kinetic ratio must be divided by a factor approximately equal to O(10) while the thermodynamic ratio is in the range $[0.1, 10]$. In the case of a mass transfer process, Φ takes values smaller or considerably smaller than 1 and is equal to 1. Thus Φ and Ξ can take values from 0.0 to ∞. To cover the situations of practical interest and to study the asymptotic behaviour, Φ and Ξ vary in this work in the range 10^{-2}–10^2.

The variation of Φ and Ξ influences or not Pr. In order to have a clear, didactical picture about the effect of Φ and Ξ on the transfer rate, it would be convenient to keep Pr constant. However, it would be unfair to ignore the situation when Pr varies concomitantly with Φ and Ξ. For this reason, both situations, i.e. Pr constant and Pr variable, are analysed in this work,

Then first task in any numerical work is to validate the codes ability to accurately reproduce published results. In the range of Re numbers used in this work, the important characteristic quantities of the flow around a cylinder are: the drag coefficient C_D, the surface vorticity and the vortex length (obviously, the vortex length can be used only at $Re = 20$). The values of C_D, maximum surface vorticity, $\omega_{s,max}$, angle of separation θ_s, and vortex length, L/d, calculated in this work are presented in Table 1. Table 1 shows that the present numerical results are in good agreement to the results published and validates the numerical solving of the Navier–Stokes equations in this work.

Unfortunately, there are no data in literature to verify the accuracy of the present heat/mass transfer computations. In order to verify the accuracy of these computations, we can use the asymptotic solutions, i.e. the internal and external problem solutions, and the solution corresponding to the case $\Phi = \Xi = 1$. The internal and external problems are one-phase, non-conjugate problems. Also, the case $\Phi = \Xi = 1$ can be viewed as one-phase, non-conjugate problem. One-phase, linear, mass/heat balance equations are simpler to solve. The accuracy and the stability of these computations are easier to verify. In addition, based on the experience of the sphere problem, one may expect that, when Φ and Ξ tend to infinity, the asymptotic Nu value of the conjugate problems tends to the Nu value corresponding to the heat/mass transfer from a cylinder with constant temperature/concentration.

The comparison of the conjugate problem solutions obtained at $\Phi \gg (\ll) 1$ and Ξ arbitrary with he external (internal) problems solutions is presented in the next paragraphs of this section. Some of the numerical experiments made to verify the accuracy of the present computations, by comparing the conjugate asymptotic Nu values obtained at $\Phi = \Xi = 100$ with those provided by published predictive relations, are presented in Table 2. Table 2 shows a good agreement between the present and published results.

The time variation of cylinder average concentration/temperature at $Re = 2$ and $RePr = \text{constant} = 100$ is plotted in Fig. 1. The curves obtained at $Re = 20$ and $RePr = \text{constant} = 100$ have the same shape and for this reason were not presented. The asymptotic values of the Nu numbers (overall and fractional) obtained at $Re = 2$, 20 and $RePr = \text{constant} = 100$ are presented in Tables 3 and 4 and plotted in Figs. 2 and 3, respectively. In each cell of Tables 3 and 4 the first line shows the asymptotic value of the internal Nu number, the second line the

Table 1
Comparison of hydrodynamic parameters with previous studies

Re	Authors	C_D	$\omega_{s,max}$	θ_s	L/d
2	Fornberg [16]	–	1.18[a]	–	–
	Tamada et al. [26]	6.65	–	–	–
	Lange et al. [1]	6.82[b]	–	–	–
	Present	6.63	1.18	–	–
20	Fornberg [16]	2.000	3.875	45.3	1.82
	He and Doolen [27]	2.152	–	42.96	1.84
	Lange et al. [1]	2.0[c]	–	–	–
	Present	1.99	3.883	43.24	1.79

[a] Evaluated from Fig. 4a of [16].
[b] Evaluated from Fig. 6 of [1].
[c] Evaluated from Fig. 7 of [1].

Table 2
Comparison of asymptotic overall Nu values with previous studies

Re	Pr	Nu	Authors
2	0.71480	1.013	Lange et al. [1]
		0.95	Present[a]
20	0.71480	2.4087	Lange et al. [1]
		2.4121	Present[a]
2	50	3.6314	Kurdyamov and Fernandez [8]
		3.88813	Kramers (Dennis et al. [2])
		3.58	Present[a]
20	5	4.9384	Kramers (Dennis et al. [2])
		4.57	Present[a]

[a] Asymptotic solution of the conjugate problem at $\Phi = \Xi = 100$.

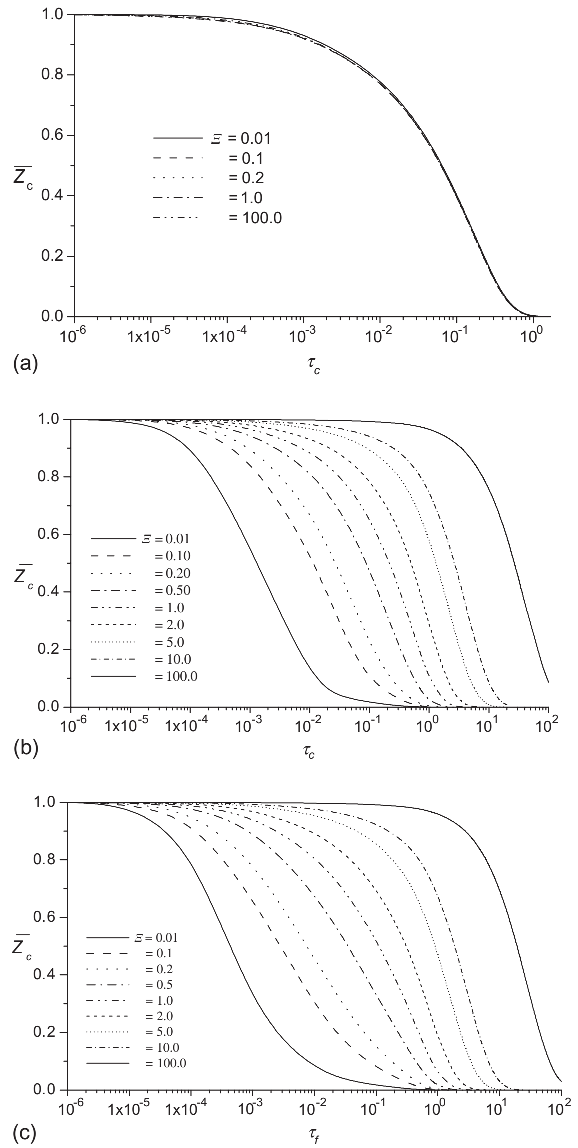

Fig. 1. Variation of the cylinder average concentration/temperature with dimensionless time at $Re = 2$ and $RePr = 100$: (a) $\Phi = 0.01$; (b) $\Phi = 1.0$; 100.

asymptotic value of the external Nu number and the third line the asymptotic overall Nu value. The presence of the superscript * in a cell indicates that the time variation of Nu does not reach a frozen asymptotic value. The values depicted in this case correspond to the integration final, when the time variation of Nu becomes small. The first and the last lines of Tables 3 and 4, i.e. the lines that correspond to $\Phi = 0$ and ∞, respectively, show the asymptotic problems solutions; the internal problem solution for $\Phi = 0$ and the external problem solution for $\Phi = \infty$.

The mass (thermal) wake phenomenon is described by the quantities defined in [11,14]. The transfer inversion point (TrIP) is the point on the cylinder surface, measured from the rear or front stagnation point, where the first negative value of the local Nu occurs. During the transfer process, the TrIP position on the cylinder surface varies in time until a steady state location is reached. The TrIP steady values are plotted in Fig. 4. Because for $\Xi \geqslant 2$ there is no mass/thermal wake, the cases $\Xi = 5$, 10, 100 are not graphically presented.

Tables 3 and 4 show a very good agreement between the solutions of the conjugate problem when $\Phi \to 0$ (∞) and the solution of the internal (external) problem. These results and those presented in Table 2 can be considered an argument for the accuracy of the present computations.

The results obtained at $Re = 2$ and $RePr = 100$ (Figs. 1, 2, 4a and Table 3) are similar to those obtained in [11,12,14] for the physical conjugate heat/mass transfer from a sphere in steady laminar flow without separation (creeping flow and moderate Re numbers smaller than the critical value). This is why we do not insist too much on these results. The main aspects that deserve to be mentioned are

- the asymptotic value of Nu_{int} increases with the increase in Φ and decreases with the increase in Ξ;
- the increase in the kinetic ratio decreases the asymptotic value of Nu_{ext} while the increase in the thermodynamic ratio increases the asymptotic value of Nu_{ext};
- for $\Xi \geqslant 1$, the variation asymptotic overall Nu versus Φ has a minimum at $\Phi = 1$; for $\Xi < 1$. the asymptotic overall Nu number decreases with the increases in the kinetic ratio;
- the influence of Φ and Ξ on the time variation of the cylinder average concentration/temperature may be related to that observed at overall Nu; note that relation (13) expresses the connection between the cylinder average concentration/temperature and the overall Nu.

Concerning the mass/thermal wake phenomenon, the following observations can be made: (1) for the same values of the kinetic and thermodynamic ratios, the cylinder TrIP values are smaller that the corresponding sphere values; (2) at cylinder, the mass/thermal wake phenomenon is not present for $\Xi \geqslant 2$.

At $Re = 20$ and $RePr = 100$, the present results and the sphere's results [12] have some common and some distinct features. The common features, that express the general characteristics of the influence of kinetic and thermodynamic ratios on the asymptotic values of the Nu numbers are

- for any Ξ value, the function Nu (Φ) has a minimum at $\Phi = 1$;

Table 3
Asymptotic Nu values at $Re = 2$ and $Re\,Pr = 100$

Φ	Ξ								
	0.01	0.1	0.2	0.5	1	2	5	10	100
0	5.78								
0.01	6.03[a]	5.81	5.81	5.81	5.81	5.81	5.81	5.81	5.81
	0.34	3.43	3.60	3.67	3.70	3.72	3.73	3.73	3.73
	5.11	5.71	5.72	5.72	5.72	5.72	5.72	5.72	5.72
0.1	7.84[a]	6.53[a]	6.18	6.07	6.05	6.04	6.04	6.04	6.04
	0.05	1.01	2.28	3.18	3.46	3.60	3.68	3.71	3.73
	0.48	4.0	4.87	5.11	5.16	5.18	5.19	5.20	5.20
0.2	7.92[a]	7.25[a]	6.74[a]	6.39	6.32	6.29	6.27	6.27	6.26
	0.05	0.60	1.44	2.74	3.24	3.49	3.64	3.69	3.73
	0.24	2.18	3.50	4.36	4.54	4.62	4.66	4.67	4.68
0.5	7.97[a]	7.70[a]	7.45[a]	7.05	6.88	6.79	6.75	6.73	6.72
	0.05	0.50	1.0	2.12	2.86	3.29	3.56	3.65	3.73
	0.10	0.90	1.59	2.65	3.12	3.34	3.46	3.50	3.53
1	7.98[a]	7.85[a]	7.72[a]	7.48	7.33	7.23	7.17	7.16	7.14
	0.05	0.46	0.89	1.85	2.61	3.12	3.48	3.60	3.71
	0.05	0.44	0.81	1.49	1.92	2.18	2.34	2.39	2.44
2	7.99[a]	7.92[a]	7.86[a]	7.74	7.64	7.57	7.52	7.51	7.49
	0.05	0.44	0.84	1.71	2.45	2.99	3.40	3.54	3.68
	0.05	0.44	0.81	1.55	2.11	2.50	2.77	2.87	2.96
5	8.0[a]	7.97[a]	7.95[a]	7.89	7.85	7.82	7.79	7.78	7.77
	0.05	0.43	0.81	1.62	2.33	2.89	3.32	3.48	3.63
	0.05	0.43	0.80	1.57	2.20	2.70	3.06	3.20	3.33
10	8.0[a]	7.99[a]	7.97	7.95	7.93	7.91	7.89	7.89	7.88
	0.05	0.43	0.79	1.58	2.28	2.84	3.28	3.45	3.61
	0.05	0.43	0.79	1.56	2.22	2.75	3.16	3.31	3.46
100	8.0[a]	8.0	8.0	7.99	7.99	7.99	7.99	7.99	7.99
	0.05	0.42	0.79	1.55	2.23	2.79	3.23	3.42	3.58
	0.05	0.42	0.79	1.55	2.23	2.79	3.23	3.41	3.58
∞	0.05[a]	0.42	0.79	1.55	2.23	2.79	3.23	3.41	3.58

[a] Unfrozen asymptotic values.

- the domain of unfrozen asymptotic values extends in the parametric space (Φ, Ξ);
- the variation of the asymptotic Nu_{ext} value versus Φ is similar to that observe at asymptotic overall Nu; the influence of Φ and Ξ on the asymptotic values of Nu_{int} is that observe at $Re = 2$ (we must mention that in [12] the influence of Φ and Ξ on the asymptotic values of the fractional Nu numbers is not discussed).

The distinct features refer especially to the mass/thermal wake phenomenon. At sphere (and cylinder in the absence of flow separation), this phenomenon occurs only in the vicinity of the rear stagnation point. In the presence of flow separation, the cylinder's wake (mass/thermal wake) has a different behaviour. At $\Xi = 1$, the mass/thermal wake is located only in the vicinity of the front stagnation point. For $\Xi < 1$ and small values of the kinetic ratio, the wake also occurs and develops in the vicinity of the front stagnation point (Fig. 5a). The increase in Φ leads to the following situation (Fig. 5b): (1) the wake appears near the rear stagnation point; (2) in time, the dimension of this wake decreases until disappearance; (3) simultaneously, another wake occurs and develops near the front stagnation point. For very small values of the thermodynamic ratio, i.e. $\Xi \leqslant 0.1$, and $\Phi \gg 1$, both wakes persist until the end of the integration time. For this reason, in Fig. 4b there are two curves for $\Xi = 0.1$ and 0.01. The line without symbol shows the steady TrIP value of the front stagnation point wake. The line with symbol refers to the wake located at the rear stagnation point.

Two elements explain the anomalous transfer observed on the front stagnation point at $Re = 20$: (1) the flow separation and (2) the variation of the local Nu

Table 4
Asymptotic Nu values at $Re = 20$ and $RePr = 100$

Φ	Ξ								
	0.01	0.1	0.2	0.5	1	2	5	10	100
0	5.78								
0.01	6.41[a]	5.81	5.81	5.81	5.81	5.81	5.81	5.81	5.81
	0.07	3.81	4.07	4.21	4.25	4.28	4.28	4.29	4.29
	3.25	5.72	5.73	5.73	5.73	5.73	5.73	5.73	5.73
0.1	7.93[a]	7.22[a]	6.39[a]	6.07[a]	6.03	6.02	6.01	6.01	6.01
	0.03	0.31	1.23	3.30	3.85	4.08	4.22	4.26	4.29
	0.22	2.20	4.23	5.13	5.22	5.25	5.27	5.27	5.27
0.2	7.96[a]	7.62[a]	7.25[a]	6.49[a]	6.29[a]	6.24	6.21	6.21	6.20
	0.03	0.24	0.60	2.24	3.43	3.90	4.15	4.22	4.29
	0.11	1.11	2.15	4.12	4.61	4.73	4.79	4.81	4.82
0.5	7.98[a]	7.85[a]	7.70[a]	7.29[a]	6.92[a]	6.75	6.67	6.65	6.64
	0.02	0.24	0.50	1.38	2.65	3.55	4.03	4.18	4.29
	0.04	0.46	0.90	2.0	3.0	3.45	3.65	3.70	3.75
1	7.99[a]	7.92[a]	7.84[a]	7.63[a]	7.39[a]	7.20	7.10	7.07	7.04
	0.03	0.24	0.48	1.24	2.32	3.34	3.98	4.18	4.35
	0.03	0.23	0.47	1.07	1.77	2.28	2.55	2.63	2.69
2	8.0[a]	7.96[a]	7.92[a]	7.81[a]	7.67[a]	7.54	7.45	7.43	7.41
	0.03	0.25	0.49	1.22	2.24	3.28	4.0	4.23	4.41
	0.03	0.24	0.48	1.13	1.96	2.70	3.15	3.29	3.40
5	8.0[a]	7.98[a]	7.97[a]	7.92[a]	7.86[a]	7.79	7.75	7.74	7.73
	0.03	0.25	0.51	1.24	2.25	3.30	4.05	4.30	4.51
	0.03	0.25	0.51	1.21	2.14	3.05	3.67	3.87	4.04
10	8.0[a]	7.99[a]	7.98[a]	7.96[a]	7.93[a]	7.89[a]	7.87	7.86	7.86
	0.03	0.26	0.52	1.26	2.27	3.32	4.07	4.32	4.55
	0.03	0.26	0.52	1.25	2.22	3.20	3.88	4.11	4.31
100	8.0[a]	8.0[a]	8.0[a]	8.0[a]	7.99[a]	7.99[a]	7.99	7.99	7.99
	0.03	0.27	0.54	1.28	2.30	3.34	4.10	4.33	4.57
	0.03	0.27	0.54	1.28	2.30	3.34	4.10	4.33	4.57
∞	0.03[a]	0.27[a]	0.54[a]	1.28[a]	2.30[a]	3.34[a]	4.10	4.33	4.57

[a] Unfrozen asymptotic values.

number on the cylinder surface. We will not insist in this discussion on the mechanism of mass/thermal wake formation. The mechanism of wake formation was analysed in [22,23]. The mass/thermal wake appears in the regions where the local Nu number takes its smallest values. At sphere with rigid interface, the local maximum Nu number is at the front stagnation point and decreases towards the rear stagnation point. At cylinder, the local Nu number takes its minimum values (local minimum values) at the two stagnation points (excepting the case of small contact times). The vortex created by the flow separation near the rear stagnation point enhances the transfer rate and displaces the wake to the other region of small Nu values, i.e. the vicinity of the front stagnation point.

At $Re = 20$ there is another problem that deserves to be discussed. Table 4 shows that for $\Xi \leqslant 1$ and $\Phi > \Phi^*$ (Φ^* depending on Ξ) the asymptotic values of the overall Nu numbers are smaller than those obtained at $Re = 2$ (Table 3). For $Re = 2$, Pr is equal to 50 while at $Re = 20$, Pr is equal to 5. In spite of the decrease in Pr by a factor equal to 10, the increase in Re by the same factor should increase the asymptotic overall Nu value. This aspect was not encountered at the sphere and contradicts the results provided by published predictive relations.

Some of the numerical experiments considering the Pr number varying simultaneously with Φ are presented in Table 5. We restricted the presentation to the data obtained at $\Phi \geqslant 1$, because we thought that the interesting situations occur especially for $\Phi \geqslant 1$. To avoid a disturbing increase in the numerical errors, the Pr number values were selected such that the product $RePr$ does not overflow 1000.

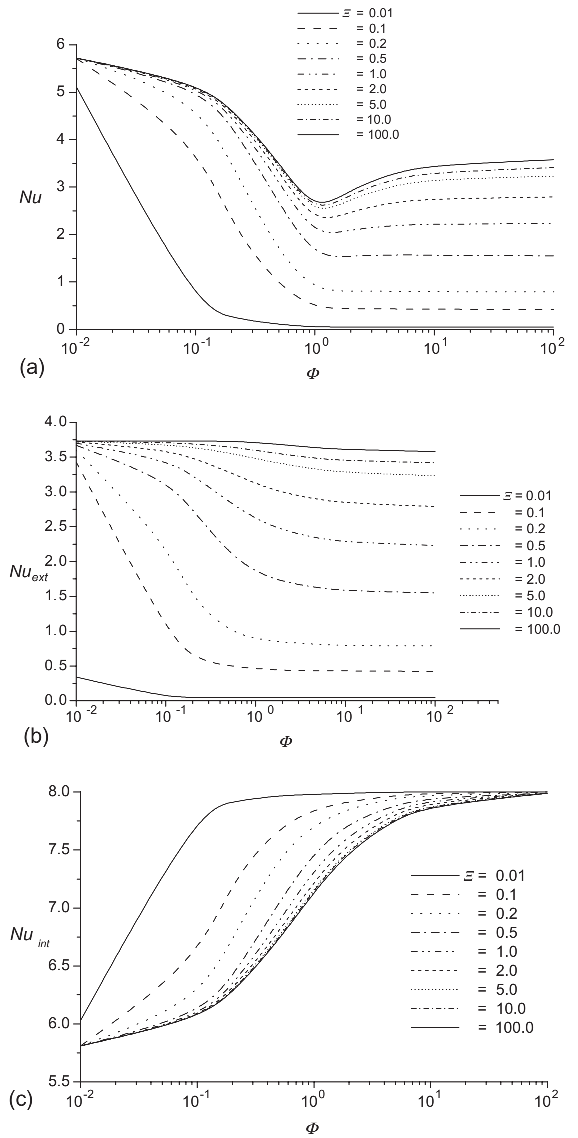

Fig. 2. Asymptotic values of the *Nu* numbers function of Φ and Ξ at $Re = 2$ and $RePr = 100$; (a) overall *Nu*; (b) external *Nu*; (c) internal *Nu*.

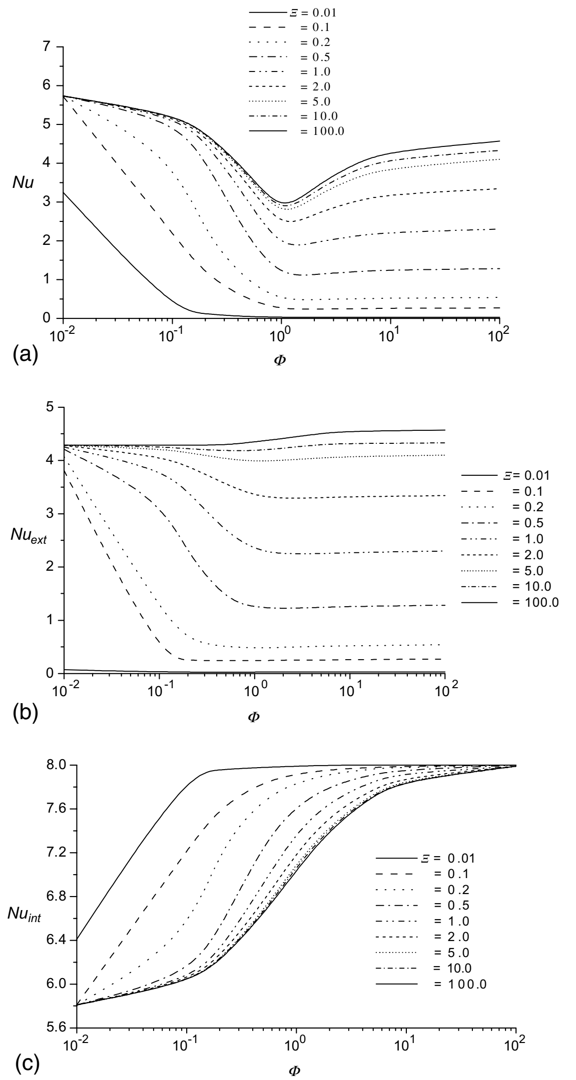

Fig. 3. Asymptotic values of the *Nu* numbers function of Φ and Ξ at $Re = 20$ and $RePr = 100$; (a) overall *Nu*; (b) external *Nu*; (c) internal *Nu*.

Table 5 shows that: (a) for a given *Re* number, the increase in *Pr* simultaneously with Φ, $\Phi \geqslant 1$, increases the overall and fractional *Nu* numbers for any value of the thermodynamic ratio; (b) for a given *Re* number and Φ value, the increase in *Pr* decreases the mass/thermal wake (this statement is based on the comparison between Table 5 and the numerical values used to plot Fig. 4). These results do not contradict the results obtained at sphere. We must also mention that the anomaly observed in Table 4, i.e. at $\Xi \leqslant 1$ some of the asymptotic *Nu* values at $Re = 20$ are smaller than those obtained at $Re = 2$, is present in Table 5.

The results presented in this section were obtained under the hypothesis of constant physical properties of the fluid. The articles dedicated to forced convection heat transfer from a circular cylinder (as example, [1,2,21]) give a special attention to the influence of the variable fluid properties on the transfer rate. Nusselt [24] proposed the following idea in order to solve in a very simple manner this problem: the computation of the non-dimensional parameters at an effective temperature of the system, $T_{\rm eff}$. In many cases, $T_{\rm eff} = T_{\rm film} = (T_{\rm c} + T_\infty)/2$. Supplementary developments of this principle can be viewed in [1]. In the case of forced convection heat transfer from a circular cylinder with constant

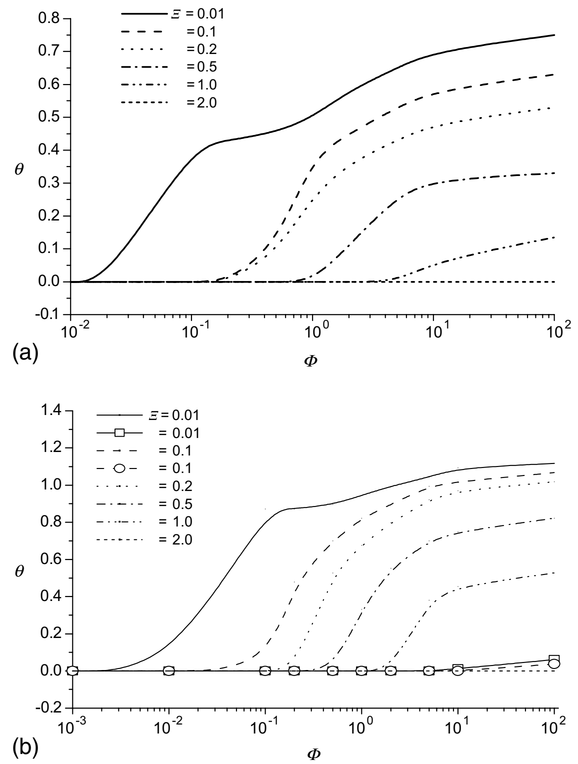

Fig. 4. The steady state position of the transfer inversion point on the cylinder surface function of Φ and Ξ at $RePr = 100$; (a) $Re = 2$; (b) $Re = 20$.

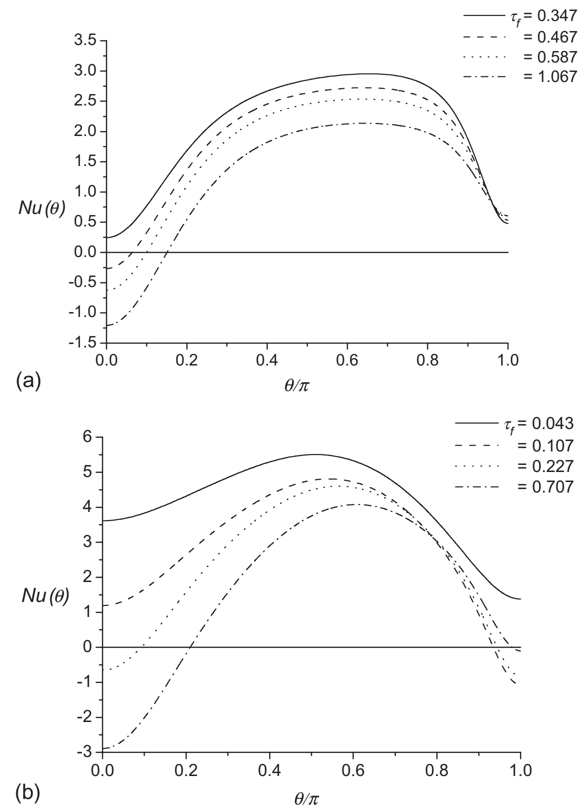

Fig. 5. Local Nu numbers at different times for $Re = 20$ and $RePr = 100$; (a) $\Phi = 2$, $\Xi = 0.5$; (b) $\Phi = 100$, $\Xi = 0.5$.

temperature, the thermal loading of the system, i.e. the ratio (T_c/T_∞), remains constant during the transfer. In the case of conjugate transfer, the cylinder temperature varies continuously during the transfer and tends to T_∞. The stabilization of the Nu number time variation and the appearance of thermal wake occur when the cylinder dimensionless average temperature becomes smaller than 0.1. Under these conditions, we think that the influence of the variable fluid properties on the asymptotic Nu values is not significant.

The main goal of any study dedicated to forced convection heat/mass transfer from a cylinder to an ambient fluid flow is to work-out a relation for the heat/mass transfer rate. Many correlation formulae can be found in [25]. Our opinion is that the present results cannot be approximated by a single relation of the type $Nu = Nu(Re, Pr, \Phi, \Xi)$. The classical method to approximate the rate of the conjugate heat/mass transfer is the interpolation relation (assimilated usually with the additivity rule)

$$\frac{1}{Nu_j} = \frac{\lambda_j}{\lambda_c}\left(\text{or } \frac{D_j H_j}{D_c}\right)\left(\frac{1}{Nu_i} + \Phi\frac{1}{Nu_e}\right), \quad j = c, f;\ H_c = 1,\ H_f = \Xi \qquad (18)$$

where Nu_i is the asymptotic Nu number of the internal problem and Nu_e is the steady Nu number calculated for the transfer from a cylinder with constant temperature/concentration. The results obtained at sphere show that relations of this type do not approximate acceptably the rate of the conjugate transfer. The present results concerning the rate of the conjugate transfer are similar to those provided by the sphere. Thus, a similar quality of the approximation given by relation (18) is expected. Some tests confirmed these assumptions. Under these conditions, we considered in this work unnecessary a comparison between the present results and those calculated with (18).

5. Conclusions

The unsteady conjugate heat/mass transfer from a circular cylinder in steady laminar crossflow was studied. Two hydrodynamic regimes were considered: flow without separation ($Re = 2$) and flow with two symmetric vortices behind the cylinder ($Re = 20$). The values of the Pr number were selected such that the product

Table 5
Numerical experiments with *Pr* variable

Ξ	Φ	*Re*	*Pr*	*Nu*	Nu_{ext}	Nu_{int}	TrIP(rad)
0.2	1	2	5	0.13[a]	0.13	7.96	0.72(r)
		20	0.5	0.14[a]	0.14	7.95	0.75(f)–0.33(r)
	2	2	10	0.24[a]	0.24	7.96	0.62(r)
		20	1	0.22[a]	0.23	7.96	0.87(f)–0.18(r)
	5	2	25	0.48[a]	0.49	7.97	0.55(r)
		20	2.5	0.38[a]	0.39	7.97	0.92(f)–0.05(r)
	10	2	50	0.79	0.79	7.97	0.48(r)
		20	5	0.52	0.52	7.98	0.97(f)
	100	2	500	3.32	3.34	8.0	0.282(r)
		20	50	1.22[a]	1.22	8.0	1.12(f)
1	1	2	5	0.54[a]	0.58	7.82	0
		20	0.5	0.62[a]	0.68	7.79	0
	2	2	10	0.90[a]	0.95	7.85	0.09(r)
		20	1	0.97[a]	1.03	7.84	0
	5	2	25	1.55	1.61	7.90	0.09(r)
		20	2.5	1.60[a]	1.67	7.89	0.36(f)
	10	2	50	2.22	2.28	7.93	0.06(r)
		20	5	2.22[a]	2.28	7.93	0.454(f)
	100	2	500	6.14	6.16	7.98	0
		20	50	5.18[a]	5.21	7.98	0.50(f)
5	1	2	5	1.18	1.40	7.59	0
		20	0.5	1.44	1.78	7.50	0
	2	2	10	1.65	1.85	7.72	0
		20	1	2.0	2.3	7.66	0
	5	2	25	2.42	2.58	7.84	0
		20	2.5	2.96	3.20	7.80	0
	10	2	50	3.16	3.28	7.89	0
		20	5	3.88	4.07	7.87	0
	100	2	500	7.19	7.23	7.98	0
		20	50	8.82	8.90	7.97	0

(f) Front stagnation point wake and (r) rear stagnation point wake.
[a] Unfrozen asymptotic value.

$Re\,Pr$ takes moderate values (from 10 to 1000). The main problem analysed is the influence of the physical properties ratios on the rate of the conjugate heat/mass transfer. The influence of these ratios on the asymptotic values of the *Nu* numbers is discussed in connection with the mass/thermal wake phenomenon.

The numerical results obtained in the previous section show that the rate of the conjugate transfer from a cylinder exhibits the same main characteristics as the rate of the conjugate transfer from a sphere. The dependence asymptotic *Nu* numbers versus Φ and Ξ follows the same rules. For a given *Re* number, the variation of the *Pr* number has the same effects as in the case of the sphere. In contradiction with the results obtained at the sphere, the variation of the asymptotic overall *Nu* versus the pair (Re, Pr) is different, function of the values of the kinetic and thermodynamic ratios. New results, that refer to the mass/thermal wake, were obtained at $Re = 20$ (flow with separation).

References

[1] C.F. Lange, F. Durst, M. Breuer, Momentum and heat transfer from cylinders in laminar crossflow at $10^{-4} \leqslant Re \leqslant 200$, Int. J. Heat Mass Transfer 41 (1998) 3409–3430.

[2] S.C.R. Dennis, J.D. Hudson, N. Smith, Steady laminar forced convection from a circular cylinder at low Reynolds numbers, Phys. Fluids 11 (1968) 933–940.

[3] D. Sucker, H. Brauer, Stationärer Stoff-und Wärmeübergang an stationär quer angeströmten Zylindern, Wärme Stoffübertragung 9 (1976) 1–12.

[4] P.C. Jain, B.S. Goel, Numerical study of unsteady laminar forced convection from a circular cylinder, ASME J. Heat Transfer 98 (1976) 303–307.

[5] G.E. Karnadiakis, Numerical simulation of forced convection heat transfer from a cylinder in crossflow, Int. J. Heat Mass Transfer 31 (1988) 107–118.

[6] C. Chen, F. Weng, Heat transfer for incompressible and compressible flows over a heated cylinder, Numer. Heat Transfer, Part A 18 (1990) 325–342.

[7] Y.T. Yang, C.K. Chen, S.R. Wu, Transient laminar convection from a circular cylinder using a body-fitted coordinate system, AIAA J. Thermophys. Heat Transfer 6 (1992) 184–188.
[8] V.N. Kurdyumov, E. Fernandez, Heat transfer from a circular cylinder al low Reynolds numbers, ASME J. Heat Transfer 120 (1998) 72–76.
[9] D.L.R. Oliver, J.N.-C. Chung, Conjugate unsteady heat transfer of a translating droplet at low Reynolds number, Int. J. Heat Mass Transfer 29 (1986) 879–887.
[10] D.L.R. Oliver, J.N.-C. Chung, Unsteady conjugate heat transfer from a translating fluid sphere at moderate Reynolds number, Int. J. Heat Mass Transfer 33 (1990) 401–408.
[11] Gh. Juncu, Conjugate unsteady heat transfer from a sphere in Stokes flow, Chem. Eng. Sci. 52 (1997) 2845–2848.
[12] Gh. Juncu, Unsteady conjugate heat transfer for a single particle and in multi-particle systems at low Reynolds numbers, Int. J. Heat Mass Transfer 41 (1998) 529–536.
[13] Gh. Juncu, The influence of the physical properties ratios on the conjugate heat transfer from a drop, Heat Mass Transfer (Wärme Stoffübertragung) 35 (1999) 251–257.
[14] Gh. Juncu, The influence of the Henry number on the conjugate mass transfer from a sphere: I—Physical mass transfer, Heat Mass Transfer (Wärme Stoffübertragung) 37 (2001) 519–530.
[15] H. Gröber, S. Erk, U. Grigull, Die Grundgesetze der Wärmeübertragung, Springer, Berlin, 1963.
[16] B. Fornberg, A numerical study of steady viscous flow past a circular cylinder, J. Fluid Mech. 98 (1980) 819–855.
[17] Gh. Juncu, R. Mihail, Numerical solution of the steady incompressible Navier–Stokes equations for the flow past a sphere by a multigrid defect correction technique, Int. J. Numer. Methods Fluids 11 (1990) 379–396.
[18] Gh. Juncu, A numerical study of steady viscous flow past a fluid sphere, Int. J. Heat Fluid Flow 20 (1999) 414–421.
[19] P.W. Hemker, A numerical study of stiff two-point boundary problems, Ph.D. Thesis, Mathematisch Centrum, Amsterdam, 1977.
[20] F. Dumouchel, J.C. Lecordier, P. Paranthoën, The effective Reynolds number of a heated cylinder, Int. J. Heat Mass Transfer 41 (1998) 1787–1794.
[21] A.P. Hatton, D.D. James, H.W. Swire, Combined forced and natural convection with low-speed air flow over horizontal cylinders, J. Fluid Mech. 42 (1970) 17–31.
[22] B. Abramzon, C. Elata, Unsteady heat transfer from a single sphere in Stokes flow, Int. J. Heat Mass Transfer 27 (1984) 687–695.
[23] Gh. Juncu, The influence of the continuous phase *Pe* numbers on thermal wake phenomenon, Heat Mass Transfer (Wärme Stoffübertragung) 34 (1998) 203–208.
[24] W. Nusselt, Das grundgesetz des wärmeüberganges, Gesund. Ing. 38 (42) (1915) 477–482.
[25] V.T. Morgan, The overall convection heat transfer from smooth circular cylinder, Adv. Heat Transfer 11 (1975) 199–264.
[26] K. Tamada, H. Miura, T. Miyagi, Low-Reynolds-number flow past a cylindrical body, J. Fluid Mech. 132 (1983) 445–455.
[27] X. He, G. Doolen, Lattice Boltzman method on curvilinear coordinates system: flow around a circular cylinder, J. Comput. Phys. 134 (1997) 306–315.